Further Studies on State Estimation of Discrete-Time Nonlinear Circuits Based on a Switching-Type Multi-Instant Fuzzy Observer

Xiangpeng Xie, *Member, IEEE*, Chang Xu, and Zhou Gu, *Member, IEEE*

***Abstract*—This brief proposes further studies on fuzzy state estimation of discrete-time nonlinear circuits. In order to give a less conservative result over the most recent one, a new switching-type multi-instant fuzzy observer is introduced and thus different groups of gain matrices can be dispatched in the light of the updated mode about the normalized fuzzy weighting functions. For each possible mode, its corresponding gain matrices are deduced by using a new time-varying balanced matrix method. Compared with the existing time-invariant balanced matrix method, much more freedom can be provided with the aid of the powerful analysis tool of multi-instant homogenous polynomials. As a result, the conservatism is reduced and better performance of fuzzy state estimation is capable of being obtained in this brief. Finally, two advantages of our proposed results are validated via the benchmark example of this field.**

***Index Terms*—State estimation, relaxed observer, nonlinear circuits, Takagi-Sugeno model, balanced matrix.**

I. Introduction

FUZZY systems are capable of coping with the problem of nonlinear synthesis and a lot of important results have been proposed for various nonlinear plants with unmeasurable system states, e.g., [1]–[2] and so on. More recently, based on the famous Takagi–Sugeno (T–S) fuzzy models [3], some difficult problems have been well solved by using model-based techniques [4]. Nevertheless, it is relevant to point out that a majority of T–S fuzzy-model-based methods are derived by using the so-called common Lyapunov functions which often cause too much conservatism [5]. To address this drawback, many relaxed techniques have been successively developed along the direction of making good use of the normalized fuzzy weighting functions(NFWFs) [6]. Particularly prominent is the development of homogenous polynomially parameter-dependent Lyapunov functions [7] so that less and less designing criteria have been acquired in [8]–[9]. Especially, a kind of time-invariant balanced matrix method has been proposed in [9] where the real-time difference characteristics of different NFWFs are compensated by using the switching-type scheme. However, the inherent limitations of its time-invariant balanced matrices produce additional conservatism, which should be solved in the future.

In practice, the vast majority of circuits' system states aren't directly available for technical or economic reasons [10]. Consequently, there exist strong practical needs for estimating them with the help of measurable system outputs [11]–[12]. As far as the fuzzy state estimation of nonlinear circuits is concerned, several results on fuzzy observers have been reported in the literature [13]–[16] while the fuzzy approximation characteristic is guaranteed in [17]. Yet, one of the common shortcomings of the above results is that their designing conditions are often with too much conservatism [18]. Early in the study, a kind of multi-instant fuzzy observer was given in [19], where both current-time and past-time NFWFs were considered by the so-called homogenous polynomially parameter-dependent way. Further, the result of [19] was relaxed by proposing a novel slack variables technique in [20]. Nowadays, it seems that more relaxed results may be achieved by using the switching-type scheme like [9] but it should be noted that the inherent limitations of its time-invariant balanced matrices should be removed as a prerequisite.

This brief aims at giving one much relaxed fuzzy state estimation of discrete-time nonlinear circuits than those existing methods of [16], [19], [20]. To achieve it, a switching-type multi-instant fuzzy observer will be introduced and thus different groups of gain matrices can be dispatched in the light of the updated mode about the NFWFs. For each possible mode, its corresponding gain matrices are deduced by using a new time-varying balanced matrix method. Compared with the existing time-invariant balanced matrix method of [9], much more freedom can be provided with the aid of the powerful analysis tool of multi-instant homogenous polynomials. As a result, the conservatism will be reduced and better performance of fuzzy state estimation will be capable of being obtained over those existing methods of [16], [19], [20]. Finally, two main advantages of our proposed results will be validated via the benchmark example of this field.

Manuscript received 28 February 2022; accepted 21 March 2022. Date of publication 23 March 2022; date of current version 1 August 2022. This work was supported in part by the National Natural Science Foundation of China under Grant 62022044, and in part by the Jiangsu Natural Science Foundation for Distinguished Young Scholars under Grant BK20190039. This brief was recommended by Associate Editor C. Wang. *(Corresponding author: Xiangpeng Xie.)*

Xiangpeng Xie and Chang Xu are with the Institute of Advanced Technology, Nanjing University of Posts and Telecommunications, Nanjing 210003, China (e-mail: xiexiangpeng1953@163.com).

Zhou Gu is with the College of Mechanical and Electronic Engineering, Nanjing Forestry University, Nanjing 210037, China.

Color versions of one or more figures in this article are available at https://doi.org/10.1109/TCSII.2022.3161686.

Digital Object Identifier 10.1109/TCSII.2022.3161686

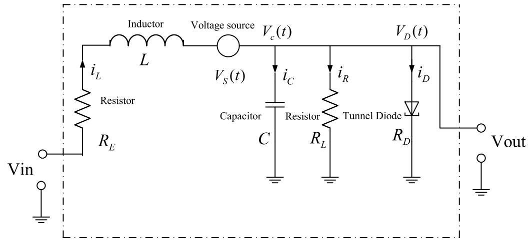

Fig. 1. The nonlinear tunnel diode circuit applied in [1].

TABLE I
THE NOMENCLATURE OF RELATED NOTATIONS

notations	meanings
r-tuple q	$q_1 \cdots q_r$
$\mathcal{K}(g)$	a set of possible q of $\sum_{1\leq i\leq r} q_i = g,\ q_i \in \mathbb{N}$
$h(t)^q$	$h_i(\nu(t))^{q_1} \times \cdots \times h_r(\nu(t))_r^{q_r}$
$h(t-1)^q$	$h_i(\nu(t-1))^{q_1} \times \cdots \times h_r(\nu(t-1))_r^{q_r}$
χ_l	$q \in \mathcal{K}(1)$, $q_l = 1$ and $q_i = 0$ with $i \neq l$
$\pi(q)$	$(q_1!) \times \cdots \times (q_r!)$
$q \pm p$	$(q_1 \pm p_1) \cdots (q_r \pm p_r)$
$q \geq p$	$q_i \geq p_i$ for $1 \leq i \leq r$
s_1, s_2, s_3	three adjustable degrees and $s_1, s_2, s_3 \in \mathbb{Z}_+$
$\mathrm{Num}(s_3)$	$\frac{(r+s_3-1)!}{s_3!(r-1)!}$

II. PROBLEM FORMULATIONS AND PRELIMINARIES

A. Problem Formulations

The following discrete-time T-S fuzzy model can be used to express nonlinear tunnel diode circuits [20]:

$$\begin{cases} \mathrm{x}(t+1) = \sum_{1\leq i\leq r} h_i(\nu(t))(A_i \mathrm{x}(t) + B_i \mathrm{u}(t)) \\ \mathrm{y}(t) = \sum_{1\leq i\leq r} h_i(\nu(t)) C_i \mathrm{x}(t) \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^{n_x}$ is its system state vector to be estimated, $u(t) \in \mathbb{R}^{n_u}$ is its input vector, $y(t) \in \mathbb{R}^{n_y}$ is its system output vector which is available at each sampling instant. $h_i(\nu(t))$ belongs to the i-th NFWF and one has $h_i(\nu(t)) \geq 0$ and $\sum_{1\leq i\leq r} h_i(\nu(t)) = 1$. For simplicity, one writes $h_j(t) = h_j(\nu(t))$ and $h_j(t-1) = h_j(\nu(t-1))$.

Remark 1: As displayed in Fig. 1, this tunnel diode circuit applied in [1] can be characterized by $C\dot{V}_C(t) = i_L(t) - \frac{V_C(t)}{R_L} - \frac{V_C(t)}{R_D}$ and $\frac{1}{R_D} = a_1 + a_2 V_D^2(t)$. Noting that $V_D(t)$ is measurable, the nonlinear term of $V_D^2(t)$ is easily expressed by the above T-S fuzzy model (1) by choosing $\nu(t) = V_D(t)$. For details, one can refer to [1].

B. Preliminaries

Related notations of homogeneous polynomials have been reviewed in Table I, which are all the same as those in [20].

Further, all the individuals of $q \in \mathcal{K}(s_3)$ with $s_3 \in \mathbb{Z}_+$ can be reordered to be $q^1, q^2, \ldots, q^{\mathrm{Num}(s_3)}$ in the descending order. Such as, if $q \in \mathcal{K}(2)$ and $r = 3$, then one gets $q^1 = 200, q^2 = 110, q^3 = 101, q^4 = 020,\ q^5 = 011,\ q^6 = 002$. And, the augmented matrix including all the matrices U_{pq} with $p, q \in \mathcal{K}(s_3)$ is defined as

$$[U_{pq}]_{\mathrm{Num}(s_3)\times \mathrm{Num}(s_3)} = \begin{bmatrix} U_{p^1 q^1} & \cdots & U_{p^1 q^{\mathrm{Num}(s_3)}} \\ \vdots & \ddots & \vdots \\ U_{p^{\mathrm{Num}(s_3)} q^1} & \cdots & U_{p^{\mathrm{Num}(s_3)} q^{\mathrm{Num}(s_3)}} \end{bmatrix}.$$

III. MAIN RESULTS

In this brief, the switching-type multi-instant fuzzy observer (2) is designed with $j \in \{1, \ldots, r\}$ as follows:

If $h_j(t) \geq h_l(t)$ with $1 \leq l \leq r, l \neq j$, then we give

$$\begin{cases} \hat{\mathrm{x}}(t+1) = A_{\nu(t)}\hat{\mathrm{x}}(t) + B_{\nu(t)}\mathrm{u}(t) + \left(Q^j_{\nu_{s_1}(t-1)\nu_{s_2}(t)}\right)^{-1} \\ \qquad \times L^j_{\nu_{s_1}(t-1)\nu_{s_2}(t)}(\mathrm{y}(t) - \hat{\mathrm{y}}(t)), \\ \hat{\mathrm{y}}(t) = C_{\nu(t)}\hat{\mathrm{x}}(t) \end{cases} \tag{2}$$

where $s_1, s_2 \in \mathbb{Z}_+$ are two adjustable degrees. And, we have

$$L_{\nu_{s_1}(t-1)\nu_{s_2}(t)} = \sum_{\binom{k'\in\mathcal{K}(s_1)}{k\in\mathcal{K}(s_2)}} \{h(t-1)^{k'} h(t)^k L^j_{k'k}\}, \tag{3}$$

$$Q_{\nu_{s_1}(t-1)\nu_{s_2}(t)} = \sum_{\binom{k'\in\mathcal{K}(s_1)}{k\in\mathcal{K}(s_2)}} \{h(t-1)^{k'} h(t)^k Q^j_{k'k}\}, \tag{4}$$

where $L^j_{k'k} \in \mathbb{R}^{n_x\times n_y}$, $Q^j_{k'k} \in \mathbb{R}^{n_x\times n_x}$ belong to gain matrices to be ascertained in Theorem 1 of the brief.

By setting $\nu(-1) = \nu(0)$, the overall estimated error system is written as:

$$\mathrm{e}(t+1) = \left(A_{\nu(t)} - \left(Q^j_{\nu_{s_1}(t-1)\nu_{s_2}(t)}\right)^{-1} L^j_{\nu_{s_1}(t-1)\nu_{s_2}(t)}\right)\mathrm{e}(t), \tag{5}$$

Remark 2: In the recent result of [20], there was only a pair of fixed gain matrices without considering any information of the updated NFWFs. As a result, its fixed gain matrices must suit for all the different cases of the updated NFWFs and thus this restriction may lead to relatively conservative results. Compared with [20], much more gain matrices are introduced in our switching type multi-instant fuzzy observer (2) in light of the actual state of the updated NFWFs. For each mode j, a pair of proprietary gain matrices $(L^j_{k'k}, Q^j_{k'k})$ can be designed for the case of $h_j(t) \geq h_l(t)$ with $1 \leq l \leq r, l \neq j$. Therefore, it does provide an important benefit of obtaining much less conservative results in this study and thus we propose the following result for solving all the observer gain matrices by means of linear matrix inequalities (LMIs).

Theorem 1: The overall estimated error system of (5) can be called to be globally asymptotically stable, if there exist gain matrices $L^j_{k'k} \in \mathbb{R}^{n_x\times n_y}$ and $Q^j_{k'k} \in \mathbb{R}^{n_x\times n_x}$ with $k' \in \mathcal{K}(s_1), k \in \mathcal{K}(s_2), j \in \{1, \ldots, r\}$; symmetric matrices $P_k \in \mathbb{R}^{n_x\times n_x}$ with $k \in \mathcal{K}(s_2)$; matrices $U_{pq} \in \mathbb{R}^{2n_x\times 2n_x}$ with $U_{pq} = (U_{qp})^T$ and $[U_{pq}]_{\mathrm{Num}(s_3)\times\mathrm{Num}(s_3)} < 0$ for all $p, q \in \mathcal{K}(s_3)$; negative definite matrices $X^{jl}_w \in \mathbb{R}^{2n_x\times 2n_x}$ with $w \in \mathcal{K}(s_4 - 1), j, l \in \{1, \ldots, r\}, l \neq j$, satisfying that the following LMIs of (6) can be guaranteed:

$$\sum_{\binom{p,q\in\mathcal{K}(s_3)}{k''\geq p+q}} \left\{ \frac{s_5!}{\pi(k''')} \frac{(s_4 - 2s_3)!}{\pi(k'' - p - q)} U_{pq} \right\}$$

$$
\begin{aligned}
&+\sum_{k''\geq\chi_j}\left\{\frac{s_5!}{\pi(k''')}\frac{(s_4-1)!}{\pi(k''-\chi_j)}\left(\sum_{1\leq l\leq r,l\neq j}X^{jl}_{k''-\chi_j}\right)\right\}\\
&-\sum_{\left(\substack{1\leq l\leq r,l\neq j\\ k''\geq\chi_l}\right)}\left\{\frac{s_5!}{\pi(k''')}\frac{(s_4-1)!}{\pi(k''-\chi_l)}X^{jl}_{k''-\chi_l}\right\}\\
&+\begin{bmatrix}\Omega^{11}_{k'''k''} & *\\ \Omega^{21}_{k'''k''} & \Omega^{22}_{k'''k''}\end{bmatrix}>0,\\
&\forall\ k'''\in\mathcal{K}(s_5),k''\in\mathcal{K}(s_4);
\end{aligned}\tag{6}
$$

where $s_4=\max\{1+s_2,2s_3\}$, $s_5=\max\{s_1,s_3\}$,

$$\Omega^{11}_{k'''k''}=\sum_{k\in\mathcal{K}(s_2),k'''\geq k}\left\{\frac{(s_5-s_2)!}{\pi(k'''-k)}\frac{s_4!}{\pi(k'')}P_k\right\},\tag{7}$$

$$
\begin{aligned}
\Omega^{21}_{k'''k''}=&\sum_{\left(\substack{k'\in\mathcal{K}(s_1),l\in\{1,\ldots,r\},\\ k\in\mathcal{K}(s_2),k''\geq k+\chi_l,k'''\geq k'}\right)}\left\{\frac{(s_5-s_1)!}{\pi(k'''-k')}\right.\\
&\left.\times\frac{(s_4-s_2-1)!}{\pi(k''-k-\chi_l)}(Q_{k'k}A_l-L_{k'k}C_l)\right\},
\end{aligned}\tag{8}
$$

$$
\begin{aligned}
\Omega^{22}_{k'''k''}=&\sum_{\left(\substack{k'\in\mathcal{K}(s_1),k'''\geq k'\\ k\in\mathcal{K}(s_2),k''\geq k}\right)}\left\{\frac{(s_5-s_1)!}{\pi(k'''-k')}\right.\\
&\left.\times\frac{(s_4-s_2)!}{\pi(k''-k)}\left(Q_{k'k}+Q^T_{k'k}\right)\right\}\\
&+\sum_{k\in\mathcal{K}(s_2),k''\geq k}\left\{\frac{s_5!}{\pi(k''')}\frac{(s_4-s_2)!}{\pi(k''-k)}P_k\right\}.
\end{aligned}\tag{9}
$$

Proof: Firstly, one can select the following Lyapunov function in this brief:

$$V(\mathrm{e}(t),\nu(t-1))=\mathrm{e}^T(t)\left(P_{\nu_{s_2}(t-1)}\right)\mathrm{e}(t),\tag{10}$$

where $P_{\nu_{s_2}(t-1)}=\sum_{k\in\mathcal{K}(s_2)}h(t-1)^kP_k$.

Hence, its one-step-forward difference $\Delta V(\mathrm{e}(t),\nu(t-1))$ is written as:

$$
\begin{aligned}
\Delta V(\mathrm{e}(t),\nu(t-1))=&\ \mathrm{e}^T(t)\left(\Upsilon^T\left(P_{\nu_{s_2}(t)}\right)\Upsilon\right)\mathrm{e}(t)\\
&-\mathrm{e}^T(t)\left(P_{\nu_{s_2}(t-1)}\right)\mathrm{e}(t)
\end{aligned}\tag{11}
$$

where $P_{\nu_{s_2}(t)}=\sum_{k\in\mathcal{K}(s_2)}\{h(t)^kP_k\}$, and

$$\Upsilon=\left(A_{\nu(t)}-\left(Q^j_{\nu_{s_1}(t-1)\nu_{s_2}(t)}\right)^{-1}L^j_{\nu_{s_1}(t-1)\nu_{s_2}(t)}C_{\nu(t)}\right).$$

Therefore, the overall estimated error system of (5) is globally asymptotically stable when one gets:

$$\Upsilon^T\left(P_{\nu_{s_2}(t)}\right)\Upsilon-P_{\nu_{s_2}(t-1)}<0.\tag{12}$$

At the same time, (12) is equivalent to the inequality of (13) by using the well-known matrix arguments:

$$\begin{bmatrix}P_{\nu_{s_2}(t-1)} & *\\ \Lambda_{21} & \Lambda_{22}\end{bmatrix}>0,\tag{13}$$

where $\Lambda_{21}=Q^j_{\nu_{s_1}(t-1)\nu_{s_2}(t)}A_{\nu(t)}-L^j_{\nu_{s_1}(t-1)\nu_{s_2}(t)}C_{\nu(t)}$, $\Lambda_{22}=Q^j_{\nu_{s_1}(t-1)\nu_{s_2}(t)}+(Q^j_{\nu_{s_1}(t-1)\nu_{s_2}(t)})^T-P_{\nu_{s_2}(t)}$.

On the other hand, if all the LMIs of (6) can be guaranteed, one has

$$
\begin{aligned}
&\sum_{\left(\substack{k''\in\mathcal{K}(s_4)\\ k'''\in\mathcal{K}(s_5)}\right)}h(t-1)^{k'''}h(t)^{k''}\mathrm{Left}(6)\\
&=\begin{bmatrix}P_{\nu_{s_2}(t-1)} & *\\ \Lambda_{21} & \Lambda_{22}\end{bmatrix}+\sum_{p,q\in\mathcal{K}(s_3)}h(t)^{(p+q)}U_{pq}\\
&\quad+\sum_{1\leq l\leq r,l\neq j}\left\{\left(h_j(t)-h_l(t)\right)\sum_{w\in\mathcal{K}(s_4-1)}h(t)^wX^{jl}_w\right\}>0,
\end{aligned}\tag{14}
$$

where we have $\Omega_{k'''k''}=\begin{bmatrix}\Omega^{11}_{k'''k''} & *\\ \Omega^{21}_{k'''k''} & \Omega^{22}_{k'''k''}\end{bmatrix}$, and $\begin{bmatrix}P_{\nu_{s_2}(t-1)} & *\\ \Lambda_{21} & \Lambda_{22}\end{bmatrix}=\sum_{\left(\substack{k''\in\mathcal{K}(s_4)\\ k'''\in\mathcal{K}(s_5)}\right)}h(t-1)^{k'''}h(t)^{k''}\Omega_{k'''k''}$, $\sum_{p,q\in\mathcal{K}(s_3)}h(t)^{(p+q)}U_{pq}=\sum_{\left(\substack{k''\in\mathcal{K}(s_4)\\ k'''\in\mathcal{K}(s_5)}\right)}h(t-1)^{k'''}h(t)^{k''}\Theta_{k'''k''}$, $\Theta_{k'''k''}=\sum_{\left(\substack{p,q\in\mathcal{K}(s_3)\\ k''\geq p+q}\right)}\{\frac{s_5!}{\pi(k''')}\frac{(s_4-2s_3)!}{\pi(k''-p-q)}U_{pq}\}$.

More importantly, recalling two prerequisites of $[U_{pq}]_{\mathrm{Num}(s_3)\times\mathrm{Num}(s_3)}<0$ and $X^{jl}_w<0$ given in this theorem, it should be pointed out that two key inequalities are always satisfied in this brief:

$$
\begin{aligned}
&\sum_{p,q\in\mathcal{K}(s_3)}h(t)^{(p+q)}U_{pq}=\xi^T\left[U_{pq}\right]_{\mathrm{Num}(s_3)\times\mathrm{Num}(s_3)}\xi\\
&<0,
\end{aligned}\tag{15}
$$

$$\left(h_j(t)-h_l(t)\right)\sum_{w\in\mathcal{K}(s_4-1)}h(t)^wX^{jl}_w\leq 0,\tag{16}$$

where one has $I\in\mathbb{R}^{2n_x\times 2n_x}$ and $\xi=\begin{pmatrix}h(t)^{p^1}I\\ \vdots\\ h(t)^{p^{\mathrm{Num}(s_3)}}I\end{pmatrix}$.

Based on (14)–(16), it is evident that (13) is guaranteed by the LMI-based conditions of (6). In other words, the overall estimated error system of (5) can be called to be globally asymptotically stable when all the LMIs of (6) hold in true. ■

Remark 3: It should be emphasized that the above three inequalities (14)–(16) play an important role in redeploying the positive and negative constraints among different tuples of k'' and k'''. As a positive result, some of the possible $\Omega_{k'''k''}$ for $k'''\in\mathcal{K}(s_5),k''\in\mathcal{K}(s_4)$ are capable of being relaxed to not be positive definite, and thus considerable freedom is introduced for reducing much conservatism of existing results [16], [19], [20]. Moreover, our proposed balanced matrices $\sum_{p,q\in\mathcal{K}(s_3)}h(t)^{(p+q)}U_{pq}$ and $\sum_{w\in\mathcal{K}(s_4-1)}h(t)^wX^{jl}_w$ are both time-varying. Compared with the well-known time-invariant balanced matrix method reported in [9], much more freedom can also be further provided with the aid of the powerful analysis tool of multi-instant homogenous polynomials via (14)–(16).

TABLE II
MAXIMAL FEASIBLE INTERVALS OF δ FOR FOUR METHODS OF [16], [19], [20] AND THE BRIEF

ϱ	1.0	1.25	1.50	1.75
[16]	[-0.4,0.7]	[-0.4,0.7]	[-0.4,0.8]	[-0.4,0.8]
[19]	[-0.4,157]	[-0.4,95]	[-0.4,64]	[-0.4,46]
[20]	[-0.4,346]	[-0.4,183]	[-0.4,113]	[-0.4,76]
The brief	[-0.4,388]	[-0.4,200]	[-0.4,120]	[-0.4,80]
rate↑	12.14%	9.29%	6.19%	5.26%

IV. SIMULATION COMPARISONS

Example: The benchmark example for (1) is considered with the same system parameters as the recent literature [20]:

$$A_1 = \begin{bmatrix} 2.5 & 1.0 \\ 0.5 & 2.0 \end{bmatrix}, \ A_2 = \begin{bmatrix} 0.5 & 0 \\ 2.5 & \varrho \end{bmatrix},$$
$$C_1 = \begin{bmatrix} \delta & 1.0 \end{bmatrix}, \ C_2 = \begin{bmatrix} 1.0 & 1.0 \end{bmatrix}.$$

It is noticed that ϱ for A_2 and δ for C_1 belong to variables to be determined via various methods. Specifically, while one lets $\varrho \in \{1.0, 1.25, 1.50, 1.75\}$, the maximal feasible interval of δ for each different ϱ can be tested by solving the LMI-based conditions of (6) iteratively. Since the method of [20] is capable of giving larger maximal feasible intervals of δ than those previous methods of [16] and [19], it only needs to compare the improved rate between the method of the brief and the method of [20] with the same adjustable degrees of (s_1, s_2, s_3), respectively. Without losing generality, if we choose $(s_1 = 1, s_2 = 2, s_3 = 2)$ which are the same as those selected in [20], then the obtained maximal feasible intervals of δ for all the methods of [16], [19], [20] and the brief have been listed in Table II. As far as the conservatism is concerned in Table II, the result of the brief is better than all the existing results of [16], [19], [20] for each fixed ϱ. That is to say, both the switching-type multi-instant fuzzy observer (2) and the usage of time-varying balanced matrix $\sum_{w\in\mathcal{K}(s_4-1)} h(t)^w X_w^{jl}$ are instrumental in reducing the conservatism over the most recent method of [20].

As found from Table II, $(\varrho = 1.75, \delta = 80)$ is out of three maximal feasible intervals of δ for existing methods of [16], [19], [20] but is within the one for Theorem 1 of the brief. Next, based on the LMIs of (6) with $s_1 = 1, s_2 = 2, s_3 = 2$, two groups of gain matrices for the switching-type multi-instant fuzzy observer (2) are obtained. For saving space, we only provide $L^j_{k'k}$ for $j = 1$, $k' \in \{10, 01\}$ and $k \in \{20, 11, 02\}$:

$$Q^j_{1020} = 10^{-3} \times \begin{bmatrix} 0.9342 & -0.4457 \\ -0.3491 & 0.1945 \end{bmatrix},$$
$$Q^j_{1011} = \begin{bmatrix} 0.0017 & -0.0005 \\ -0.0004 & 0.0001 \end{bmatrix},$$
$$Q^j_{1002} = \begin{bmatrix} 0.0011 & -0.0001 \\ -0.0001 & 0.0000 \end{bmatrix},$$
$$Q^j_{0120} = 10^{-3} \times \begin{bmatrix} 0.7290 & -0.3420 \\ -0.2755 & 0.1412 \end{bmatrix},$$
$$Q^j_{0111} = \begin{bmatrix} 0.0017 & -0.0006 \\ -0.0004 & 0.0001 \end{bmatrix},$$
$$Q^j_{0102} = \begin{bmatrix} 0.0011 & -0.0000 \\ -0.0001 & 0.0000 \end{bmatrix}.$$

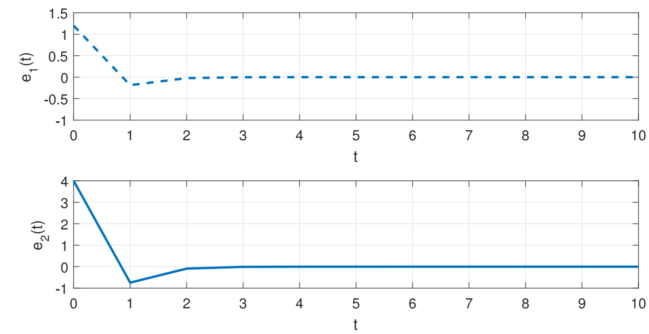

Fig. 2. Two responses of $e_1(t)$ and $e_2(t)$.

Then, we can select $x(0) = (0.6, \ -1.0)^T$ and $\hat{x}(0) = (-0.6, \ -5.0)^T$, and two responses of $e_1(t)$ and $e_2(t)$ quickly converge to the zero equilibrium point by the aid of the switching-type multi-instant fuzzy observer (2), see Fig. 2 for details.

Next, it has been reported in [20] that another parameter configuration of $(\varrho = 2, \delta = 38)$ is within the maximal feasible interval of δ for [19] and [20] but is out of the maximal feasible interval of δ for [16]. So, the latter three methods ([19], [20], and Theorem 1 of the brief, respectively) are available for $(\varrho = 2, \delta = 38)$. For this case, the performance levels of fuzzy state estimation should be compared with each other for [19], [20], and Theorem 1 of the brief. Based on the LMIs of (6) with $s_1 = 1, s_2 = 2, s_3 = 2$, two groups of gain matrices for the switching-type multi-instant fuzzy observer (2) are obtained. For saving space, we only provide $Q^j_{k'k}$ for $j = 1$, $k' \in \{10, 01\}$ and $k \in \{20, 11, 02\}$:

$$Q^j_{1020} = \begin{bmatrix} 6.6108 & -3.0495 \\ -2.1690 & 1.2306 \end{bmatrix},$$
$$Q^j_{1011} = \begin{bmatrix} 10.1063 & -2.4994 \\ -2.4573 & 1.0580 \end{bmatrix},$$
$$Q^j_{1002} = \begin{bmatrix} 8.0056 & -0.3147 \\ -1.7480 & 0.2046 \end{bmatrix},$$
$$Q^j_{0120} = \begin{bmatrix} 5.9566 & -2.7885 \\ -2.0201 & 1.1117 \end{bmatrix},$$
$$Q^j_{0111} = \begin{bmatrix} 10.3036 & -2.7414 \\ -2.5579 & 0.9999 \end{bmatrix},$$
$$Q^j_{0102} = \begin{bmatrix} 7.9349 & -0.2780 \\ -1.8551 & 0.1511 \end{bmatrix}.$$

Finally, we can select the same initial conditions as above, and thus three kinds of different methods (i.e., [19], [20] and Theorem 1 of the brief) are employed to accomplish the same task of fuzzy state estimation in order to facilitate comparison. Indeed, the indicator of estimation accuracy can be measured by $J(t) = \sum_{l=1}^{t} e^T(l)e(l)$. Here, the comparisons of different $J(t)$ for [19], [20] and the brief have been provided in Fig. 3. There are two advantages obtained in the brief by observing Fig. 3: 1) $J(t)$ for the brief is the smallest one and this means that a better estimation accuracy has been acquired in the brief; 2) The curve of $J(t)$ for the brief rapidly approaches

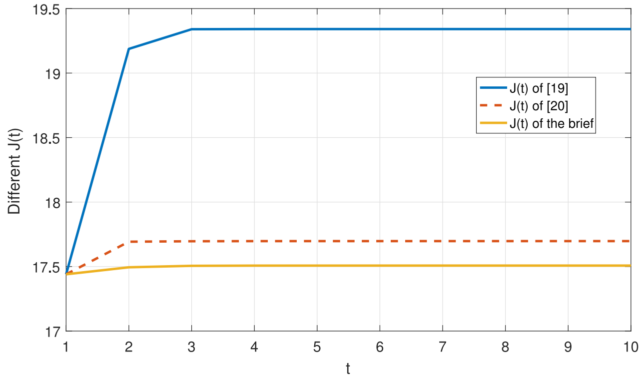

Fig. 3. The comparisons of different $J(t)$ for [19], [20] and the brief.

its peak and this means that the method of the brief has a faster response than the other two methods of [19] and [20].

V. Conclusion

This brief has proposed relaxed fuzzy state estimation of discrete-time nonlinear circuits. For proposing better results over the most recent one of [20], a switching-type multi-instant fuzzy observer has been introduced and thus different groups of gain matrices can be dispatched in light of the updated mode about the NFWFs. For each possible mode, its corresponding gain matrices have been deduced by using our time-varying balanced matrix method. Compared with the existing time-invariant balanced matrix method, much more freedom can be provided with the aid of the powerful analysis tool of multi-instant homogenous polynomials. As a result, the conservatism has been reduced and better performance of fuzzy state estimation has been obtained in this brief. Finally, two advantages of our proposed results have been validated via the benchmark example of this field. In our future research, how to develop much better/robust state estimations of discrete-time nonlinear circuits subject to malicious attacks (i.e., the system output $y(t)$ may be occasionally tampered or even discarded during the sampling interval) will be another interesting topic to be investigated.

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